Concept of Bulk Temperature

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Temperature is a concept when attempted for a definition can fool most of us (Energy is another one). If we want a quick and easy definition we probably would settle for temperature is a property that gives a measure of the degree of hotness or coldness of a body. But this post is about an extended concept: The Bulk temperature or Adiabatic Mixing Cup temperature

The concept of bulk temperature is used in internal forced convection situations like pipe flows and heat exchangers. It is a compromise between Thermodynamics and Forced Convection Heat Transfer. From thermodynamics, for a static homogeneous fluid system made of a single chemical constituent, we can write the temperature as

$$T = \frac{I}{C} \tag{1}$$

the ratio between total enthalpy, I (= mass (m, kg) x specific enthalpy (i, J/kg)), and heat capacity, C (= mass (m, kg) x specific heat capacity (c, J/kg.K)), of the system.

A little digression about the term heat capacity; It gives the erroneous impression of heat being contained in a system; after so much literature-usage it is now too late to change it to the correct internal energy capacity, hence we stick with the old oxymoron.

Now, consider the steady forced convection channel flow shown below, with constant heat flux (q", W/m2) as shown in Fig. 1. Observe in Fig. 1, the
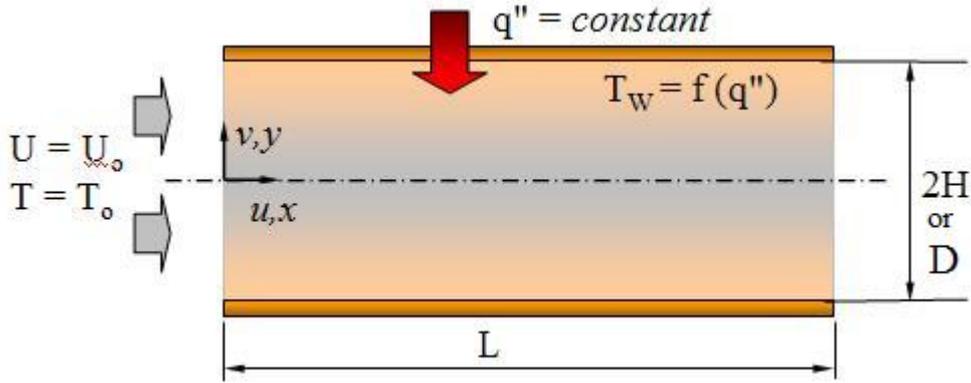

Fig. 1. A two dimensional schematic of a forced convection heat transfer channel flow configuration

schematic is an idealization of a mid-plane in z-direction (perpendicular to the plane of the paper or computer screen). Hence the figure could represent a parallel (top and bottom) plate confined channel flow or a parallelepiped pipe flow (cross section being a rectangle).

Here, both the enthalpy and heat capacity become dependent on the rate of flow, but, if we treat I and C as local quantities, Eq. (1) is still valid for every point within the channel. To retain the notion of temperature in this situation, a bulk flow model is employed. This is because the temperature inside the channel at any point can be of different values, i.e. $T = T(x, y)$. Since the fluid in the channel is being heated from the walls, one can expect the fluid near the wall to be hotter than the one near the axis (middle of the channel).

This means, at any cross section along x, one can expect a temperature "profile" for the fluid that is very high near the wall and close to the inlet temperature (but can be greater than it) near the axis. This situation should be true for any cross section along the x direction (from left to right in Fig. 1), irrespective of the wall is kept at constant temperature (higher from that of the fluid) or at constant heat flux (as shown in fig. 1). An explanatory picture is given in Fig. 2.

So, it is obvious now that the situation cannot be treated even for steady state, as a closed thermodynamics system represented by a single temperature. It is convenient now to treat the channel flow as a thermodynamic analysis (energy balance) of an open system with the definition in Eq. (1) recast for a cross section of the channel, as

$$T_b = \frac{\text{rate of flow of enthalpy through a cross section}}{\text{rate of flow of heat-capacity through a cross section}} \qquad (2)$$
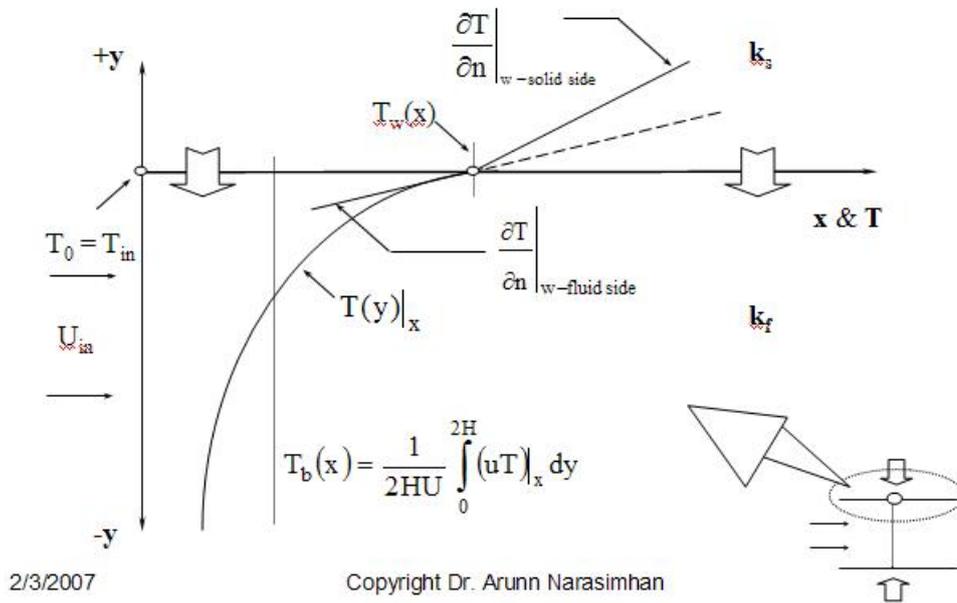
Fig. 2. Snapshot of temperature profile at a location x in forced convection channel flow configuration

In Eq. (2), we are accounting for the (cross sectional) spatial variation of temperature within a (thermodynamic) system (i.e. the channel), a better representation of the system temperature, when the system is treated as a whole.

Hence we can do a thought experiment to measure the bulk temperature. If we slice the channel at any cross section, at any location along its length, and allow the fluid to pour itself into an adiabatic (thermally insulated from the surrounding) cup, the well-mixed fluid in the cup itself can be treated as a static homogeneous fluid system. Equation (1) holds for this system, and the evaluated temperature of the cup (using the thermometer shown in Fig. 3) is called the adiabatic mixing cup temperature, another name for the bulk temperature.

But one cannot always translate this thought experiment into practice. For devising a practical way to measure this bulk temperature, we need some more things to be defined - for us to measure.

The mass flow rate across a cross sectional element in the channel as shown in Fig. 2, is given as $\rho u A dA$, where ρ and u are the density and local velocity ($u = u(x, y)$) of the flowing fluid respectively, A is the flow cross section area of the channel at a particular x location. Similarly, the enthalpy flow rate across a cross section at a given location x for the situation in Fig. 2, is given
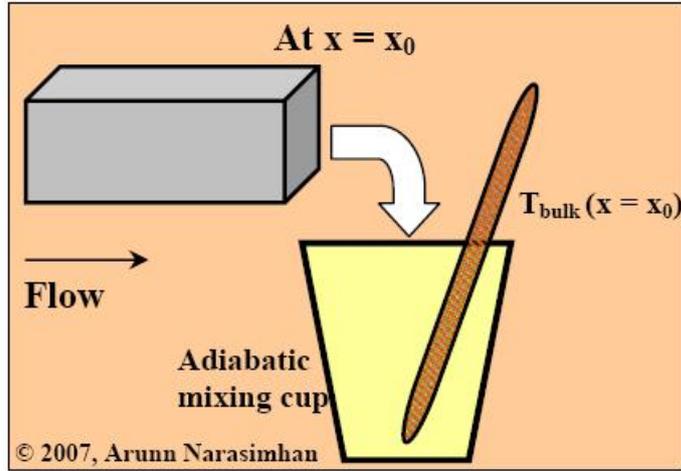
Fig. 3. The concept of the bulk temperature - a thought experiment

as $\rho u A d A c_P T$, where c_P (small 'c' and capital P) is the specific heat of the fluid measured at constant pressure and T is the local temperature ($T = T(x, y)$). Notice here enthalpy is replaced only by $c_P T$, under the assumption of negligible local pressure changes. If this is not true we have to include a term with pressure change and beta, the volume expansivity of the fluid.

Equipped with the above definitions, we can recast Eq. (2) in a general shape as,

$$T_b = \frac{\int_A \rho u c_P T dA}{\int_A \rho u c_P dA} \cdots (3) \tag{3}$$

Notice the use of c¡sub¿P¡/sub¿ in Eq. (3) generalizes the concept to include compressible fluid flow as well. For incompressible fluids (such as water), c¡sub¿P¡/sub¿ can be replaced with their respective specific heat (c). With c¡sub¿P¡/sub¿ treated as constant and the replacement of the area integral of local velocity u by UA, the channel cross section averaged velocity times the cross sectional area, Eq. (3) reduces to

$$T_b = \frac{1}{UA} \int_A u T dA \tag{4}$$

the generalized definition of bulk temperature. For the two dimensional channel in Fig. 1, Eq. (4) takes the form

$$T_b(x) = \frac{1}{2HU} \int_0^{2H} (uT)_x dy \tag{5}$$

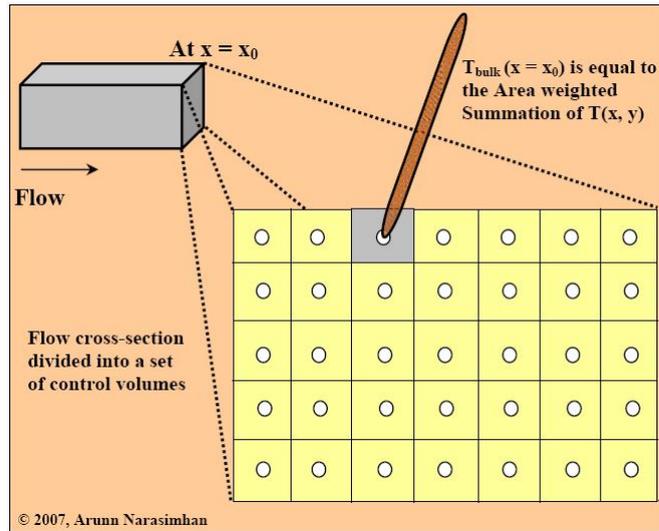This temperature defined in Eq. (5) is applicable while the fluid is flowing

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Fig. 4. Experimental determination of bulk temperature in a steady channel flow, forced convection situation

inside the channel and if the thought experiment of Fig. 3 could be performed, would equal the temperature measured in Eq. (1). Further, as a concept, it is valid for both laminar and turbulent internal flows.

However, this definition in Eq. (5) also makes it difficult to be measured in an experiment accurately. But an approximation can be done using the method suggested in the Fig. 4 below.

Each white dot is the place where actual temperature and local velocity measurements are made while the flow persists. Such local values are approximated to represent the temperature for a control volume of the fluid region that surrounds those points (In Fig. 4, represented by the rectangles formed by the grid). An area weighted summation of all such temperature values should give an answer that is close to the RHS of Eq. (5). The fineness or coarseness of the grid in Fig. 4 determines the accuracy of the bulk temperature measurement.

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