Concept of Viscosity

Arunn Narasimhan [1]

Associate Professor,
Department of Mechanical Engineering,
Indian Institute of Technology Madras,
Chennai, Tamil Nadu 600036
Email: arunn@iitm.ac.in
Web `http://www.nonoscience.info/`

All matter that obeys conservation laws of physics [1 - see endnote], offer resistance to change of their shape. A fluid is differentiated from a solid by its *mode of resistance* to the impressed force that purports to change the shape. Both liquids and gases are classified as fluids due to the similarity in their mode of resistance.

When a normal stress [2] σ (i.e. force applied per unit area normal to the boundary of matter) prevail, both fluids and solids respond (resist) identically, by undergoing a strain ϵ (deformation), obeying the mathematical statement $\sigma = E\epsilon$, where E is the modulus of elasticity. The relationship is called Hooke's Law [3].

However, when a //shear// stress $latex\tau$ (i.e. force applied per unit area tangential to the boundary of matter) acts, a solid responds with a finite angular (tangential) strain, whereas a fluid strains (deforms) continuously (in time) as long as the shear stress prevails. The stresses in an elastic solid are proportional to the finite deformation while in the fluid they are proportional to the rate of deformation.

At the beginning of section 9 of Book 2 of his Principia [4], Sir Isaac Newton (1642-1727) hypothesized [5] this behaviour of a fluid as "the resistance arising from the friction (or *lack - or want - of lubricity or slipperiness*) of the parts of a fluid, other things being equal, is proportional to the velocity with which the parts of the fluid are separated".

Today, the above hypothesis is restated as the shear stress inside a fluid being proportional to the velocity gradient, with viscosity being the constant of proportionality. This physical interpretation of viscosity as a constitutive

---

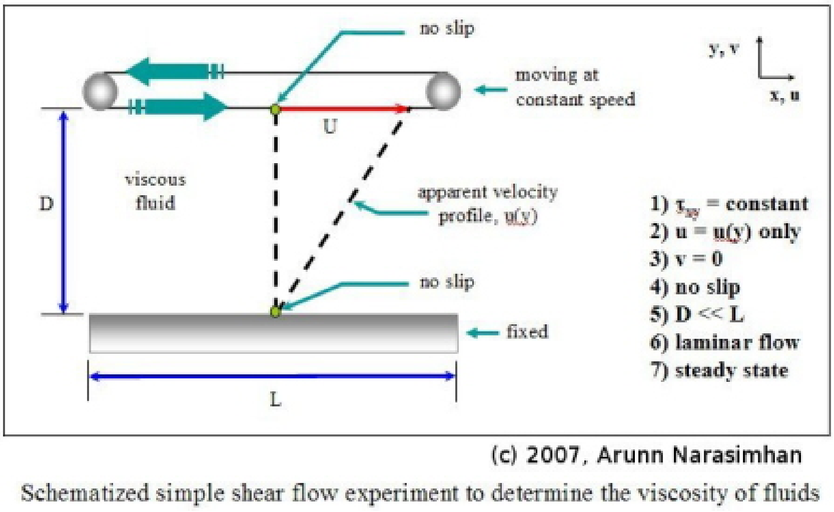

Fig. 1. schematic of a thought experiment to determine dynamic viscosity

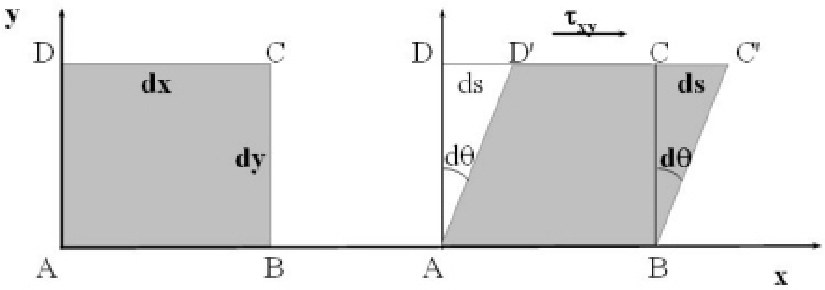

Fig. 2. elemental fluid region subject to shear

property of any fluid was due to the works of Navier (1822), Poisson (1831), St. Venant (1843), and Stokes (1845), almost two centuries after the Principia.

To introduce viscosity more rigorously, let us retrace the widely used example of a fluid between two parallel surfaces, as shown in Fig. 1.

Now, consider a square fluid element, as shown in Fig. 2, near the bottom wall of Fig. 1.

When subjected to a constant shear stress on the top surface (which in 2D, is seen here as a line), the element will deform continuously in time, with (an assumed) constant strain rate ϵ_{xy} equivalent to the angle $d\theta$ that AD makes while straining to AD', over a certain time dt.

To put in simple mathematics, for DD' (and CC') being small, from Fig. 2,

$$ds = dy \times d\theta \quad \therefore \; d\theta = ds/dy \tag{1}$$

and hence for time dt, the strain rate can be expressed as

$$\epsilon_{xy} = \frac{d\theta}{dt} = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{du}{dy} \tag{2}$$

Assuming the shear stress to be linearly related to the strain rate, for our fluid element we can write

$$\tau_{xy} \sim \epsilon_{xy} \;\therefore\; \tau_{xy} = \mu \frac{du}{dy} \tag{3}$$

where the proportionality constant is μ and is known as the coefficient of viscosity, introduced by Navier (1822). It is also known as the shear viscosity or dynamic viscosity.

The above relation can essentially be carried over to Fig. 1, for the entire fluid experiencing the shear stress between the parallel surfaces. This can be visually perceived by considering another element similar to $ABCD$ sitting on top of $ABC^{'}D^{'}$ and offering identical strain rate $latexd\epsilon$ in time dt, for a identical τ_{xy} on its top surface and by repeating the building procedure until we reach the top wall of Fig. 1. Then, the extended elemental line $AD^{'}$ will match the slanted dashed line in Fig. 1, that denotes the apparent velocity profile. With this, we essentially have constructed a careful experiment to determine the viscosity of the fluid subjected to simple shear [6]

The experiment, when performed with different fluids, with the restrictions 1 through 7 shown in the inset of Fig. 1, would result in the applied shear stress (in Fig. 1, τ_{xy} is the shear stress applied in the x direction, and normal to the y direction, i.e. on the x-z plane of the fluid) being a unique function of ϵ_{xy}, the resulting strain rate.

That is, for simple fluids such as water, air etc. where the functional relationship is indeed linear, we can write

$$\tau_{xy} \sim \epsilon_{xy} (or) \tau_{xy} = \mu \frac{U}{D} = \mu \frac{0-U}{0-D} (or) \tau_{xy} = \mu \frac{du}{dy} \tag{4}$$

According to Eq. (4), a plot between τ_{xy} and ϵ_{xy} would result in a straight line passing through the origin, the slope of which indicating the viscosity μ of the tested fluid.

When the plot yields a curve other than the straight line, the fluid tested is said to be exhibiting non-Newtonian behaviour. The subject of non-Newtonian fluid flow is subsumed under a larger endeavor called rheology, the science of deformation and flow, which includes the study of the mechanical properties of a cornucopia of materials from gases and liquids to asphalt and clay. In this chapter we deal only with fluids exhibiting Newtonian viscosity behaviour.

A good reference point to explore more historical evolution of such fluid mechanics concepts is Nemenyi (1962) and Tokaty (1971).

Endnotes

(1) by this we exclude from our present discussion, plasma-like state of matter such as bio-plasma, the aurorae or corona discharge etc. wherein (instead of distinct solid, liquid, gas) other states may exist where the conservation laws may not hold. See Granger (1995) for the origin of this note.

(2) For a solid and fluid at rest, stress, the mechanical pressure and the thermodynamic pressure are identical while for a fluid in motion, stress and pressure are different and in principle, all three of them can have different values. See chapter 2, section 2.4-3, of White (1991), for more details.

(3) named after the British scientist Robert Hooke (1635 -1703). He was originally employed by Robert Boyle in 1655 to work on the Boylean air pump and five years later discovered this law of elasticity. He was also the first man to state publicly all matter expands when heated and air is composed of particles separated by large distances.

(4) we consult here, two English translations that covered material from all of the three editions of the Principia published respectively in 1687, 1713 and 1729 (see the Reference section, for details).

(5) Although Newton probably had a strong reason for putting it as an hypothesis, the concept of viscosity as a fundamental property of any fluid nevertheless emerges only from that hypothesis. In the subsequent part of section 9, Newton then uses this hypothesis to explain the motions generated by an infinitely long cylinder and sphere rotating in a fluid, as a means to propound his theory of vortices.

(6) more detailed analysis of viscosity, based on Stokes (1845) method, can be obtained from text books like Prandtl and Teitjens (1934) and White (1991).

References

(1) Granger,R. A., 1995 Fluid Mechanics, Dover Pub., page 41.

(2) Navier, C. L. M. H., 1822 Mmoire sur les Lois du Mouvement des Fluides. Mm. de l' Acad. des Sciences, 6, 389.

(3) Nemnyi, P. F., 1962 The Main Concepts and Ideas of Fluid Dynamics in their Historical Development. Archives for History of Exact Sciences, 2,

52-86.
(4) Newton, Sir. I., 1687(1st ed., 1713, 2nd ed. and 1726, 3rd ed.) Philosophiae Naturalis Principia Mathematica. 1) translated by A. Motte, Prometheus Books, New York, 1995; originally published as: Newton's Principia. 1st American ed., New York, 1848. 2) Cajori, F., Newton's Principia (Motte's translation revised). Uty. of Cal. press, Berkeley, CA, 1946.
(5) Poisson, S. D., 1831 Mmoire sur les quations gnrales de l'quilibre et du Mouvement des Corps solides lastiques et des Fluides. J. de l'cole Poly. 13, 1.
(6) Prandtl, L. and Tietjens, O. A., 1934 Fundamentals of Hydro- and Aeromechanics. Dover Pub. Inc., New York.
(7) Stokes, G. G., 1851 On the Effect of the Internal Friction of Fluids on the Motion of Pendulums. Cambridge Phil. Trans. 9, 8-106.
(8) St. Venant, 1843 Comptes Rendus, 17, p 1240.
(9) Tokaty, G. A., 1971 A History and Philosophy of Fluid Mechanics. Dover Pub. Inc., New York.
(10) White, 1991 Viscous Fluid Flow 2nd edition. McGraw Hill Publishers, New York.